The Energetics of Project Chiron

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The Newtonian mechanics of elliptical orbits gives a simple formula for the energy of an orbit. By convention, the energy of a pair of bodies is zero at that conceptual point where they go from being bound by gravity to being unbound by gravity. So all bound orbits have negative energy, and the tighter the orbit the more negative the energy gets.

$$\frac{E}{m} = -\frac{GM}{2a} \tag{1}$$

Where E is the orbital energy, m is the mass of the smaller body, G is the universal gravitational constant, M is the mass of the larger body, usually the sun, and a is the semi-major axis of the orbit, which reduces to the radius of the orbit for circular orbits.

$$\Delta E = -GMm(\frac{1}{2a_1} - \frac{1}{2a_2}) \tag{2}$$

The mass of Chiron is not known very well, but if we adopt a ballpark water rich density typical of outer solar system moonlets of $\rho = 1500$ kg m^{-3} and a radius of 110 km then its approximate mass is $m = \rho\frac{4}{3}\pi R^3 = 8.4 \times 10^{18}$ kg.

The orbital parameters for Chiron, on the other hand, are very well known. It's semi-major axis is $a_1 = 13.637$ A.U., where one A.U. is the the earth's semimajor axis: $a_2 = 1$ A.U. $= 1.496 \times 10^{11}$ m. The mass of the sun is $M = 1.99 \times 10^{30}$ kg, and the gravitational constant is $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The SI unit for energy is the Joule. Plugging all that into equation 2, we obtain

$$\Delta E = -GMm(\frac{1}{2a_1} - \frac{1}{2a_2}) = 3.5 \times 10^{27} \text{ J} \tag{3}$$

If, instead, we chose to extract a similar mass from the surface of the earth and lift it to the moon's orbit, the same equation applies, substituting the mass of the earth for M and the earth-centered distances for a_1 and a_2. We find an energy expenditure about a factor of ten less: $\Delta E = 2.6 \times 10^{26}$. We argue elsewhere, however, that the environmental imact to earth would be large.

Reaction mass calculation

The relation between thrust and mass consumption rate is given by

$$F = v_e\dot{m} \tag{4}$$

where F is the thrust in Newtons, v_e is the velocity of the particle stream coming out of the engine, and $\dot{m}$ is the mass consumption rate in kg s^{-1}.

The other relevant equation is

$$F = 2\eta P/v_e \tag{5}$$

where η is a dimensionless efficiency factor between zero and one that depends on engineering design, and P is the power consumed by the engine. Setting these two equations equal to one another

$$\dot{m} = 2\eta P / v_e^2 \tag{6}$$

A "VASIMR" design ion thruster has, very roughly, $v_e = 100,000$ m s^{-1}. If we note that $\Delta E = \Delta t P$ if the power is constant with time, then

$$\Delta m = \Delta t \ \dot{m} = 2\eta \Delta E / v_e^2 \tag{7}$$

And we can calculate the amount of mass lost for our test case for moving Chiron from the outer solar system to earth's location: $\Delta M = 3.5 \times 10^{17}$ kg, or about 4% of the mass of Chiron. In terms of reaction mass, the journey to the inner solar system is not costly, and delivers all of the mass, even if it entirely rocket powered without cleverness from planetary gravitational assists.

If the power is supplied entirely by photovoltaic panels (probably a poor choice when compared to fusion), it may take a while for the sun to power the journey. Given two efficiency factors, one for the engines and one for the solar panels, the useable power is

$$P = \eta_{panel} \eta_{engine} A L_\odot / d^2 \tag{8}$$

where d is the distance from the sun to Chiron, A is the area of panels exposed to the sunlight, and $L_\odot$ is the luminous power of the sun in W s^{-1}. Chiron starts at 13.6 A.U. and ends up at 1 A.U., so the average distance is around 7 A.U., and the luminosity of the sun is 3.846×10^{26} W s^{-1}.

We have freedom to choose the amount of surface area covered by panels, so let us cover a 40 km by 40 km area, giving a surface area of $A = 1.6 \times 10^9$ m^2. If we choose $\eta_1 = \eta_2 = 0.7$, then the useable power is $P = 2.7 \times 10^{11}$ W. That looks like a big number, but if you look at how long it would take to move Chiron in its orbit, that timescale is

$$\tau = E/P \approx 400 \text{ million years.} \tag{9}$$

That is too long a time! A human timescale of 100 years would be more reasonable, but that requires a power output of

$$P = E/\tau \approx 10^{18} \text{ W} \tag{10}$$

That is about 80,000 times the world's current energy consumption rate of 1.2×10^{13} W. To move Chiron to the inner solar system seems to require fusion energy and new technology.